and 28.9 hr, respectively (Fig. 3a). If the drag horsepower were increased by 3000 hp, causing increases in (DHP/W) of 17, 38, 17, and 32% for the four designs, their endurances would be reduced by 15, 32, 16, and 29%, respectively.

Sprint/Drift Performance Mode

The performance capability of the 40,000-SHP family (25% m_{T0} criterion) was also calculated for a mode of operation in which the hydrofoil would alternately sprint at speed V_S and then drift at speed V_D (hullborne, with its engine virtually idling). Let the fraction of time sprinting be τ_S, and the fraction of time drifting be τ_D ($\tau_S + \tau_D = 1$). The sprint/drift time split depends upon the assumed average speed for the cycle V_{avg} (assumed to be 20 knots), the sprint speed V_S (variable), and the drift speed V_D (assumed to be 10 knots). Equations (2) and (3) become:

$$E_H = \frac{2240\ (0.9\ W_F)}{(\text{SFC})(W - 0.45\ W_F)[(\tau_S/\eta_S)(\text{DHP}/W)_S + (\tau_D/\eta_D)(\text{DHP}/W)_D]} \qquad (4)$$

$$R = E_H(\tau_S V_S + \tau_D V_D) \qquad (\tau_S + \tau_D = 1) \qquad (5)$$

where

$$\tau_D = (V_S - V_{avg})/(V_S - V_D) = (V_S - 20)/(V_S - 10) \qquad (6)$$

Figure 4 compares results from Eq. (4) with the constant-speed results from Eq. (2). Endurances are significantly increased by sprint/drift operation, and maximum endurance is attainable at a higher (sprint) speed than in the constant-speed case (e.g., 34 vs 29 knots for 800/PR).

Concluding Remarks

The relationships among first-order hydrofoil design parameters and design criteria have been examined in a cursory manner for large hydrofoil configurations. The results in Table 1 and Fig. 3 can be used to select first-order designs for various characteristics. For example, for maximum endurance with a 25% margin on thrust at takeoff speed (Fig. 3a), a configuration with 800-psf foil loading and propeller propulsion would be chosen, but it would be limited in maximum speed to 36 knots (Table 1), which is low compared to other configurations. A cursory look at the sprint/drift mode of operation has indicated that it can significantly improve endurance capability.

The analysis should be expanded to address effects of additional parameters of interest and employ refined design and performance equations. The sensitivities of the speed, endurance, and range results to operational speed mode, incremental drag horsepower, payload fraction, and SFC variations with powerplant size and vehicle speed, should be explored.

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Analysis of Gas Bubble-Liquid Coflow Systems

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Nomenclature

A = flow cross section area
D = bubble diameter
E = entrainment rate
F_s = shear force on nozzle walls
f = viscous correction
g = acceleration of gravity
h_s = bubble surface heat-transfer coefficient
k = added mass coefficient
$\dot{m}$ = mass flow rate
Nu = Nusselt number
Pr = Prandtl number
$\dot{q}$ = heat flux
R = gas constant
T = static temperature
V_B = bubble volume
α = bubble volume fraction
κ = thermal conductivity
σ = surface tension coefficient
$(\ )_B$ = bubble
$(\ )_L$ = liquid

Introduction

DURING the course of an investigation of trajectories of thermal effluents, the aeration of the jet water to enhance turbulent mixing was proposed. The present one-dimensional analysis was undertaken to provide a better understanding of the effects of air bubbles on thermal jets, and to provide a foundation for developing an experimental study of thermal jets with aeration. Furthermore, there is presently no general one-dimensional analysis of a bubble-liquid coflow available in the literature, a situation that seems surprising considering the uses to which these flows are put. In a recent study by Cederwall and Ditmars[1] a related problem of destratification of a lake by an air-bubble plume was considered. They used a two-dimensional analysis in which the bubble plume is assumed to act like a jet, with a simple jetlike entrainment model, that produced successful results. Other work on a water-air bubble coflow system has been carried out by Muir and Eichhorn[2] in a study of air-augmented water propulsion systems. They investigated nozzle flows with the intent of increasing thrust levels by making use of the slip velocity of the bubbles. That is, augmented thrust can be obtained since the bubbles move faster than the water in an accelerated flow. Another area in which air-water coflow is used is that of activated sludge aeration in which air-bubble plumes are employed to promote mixing and to provide oxygen for the micro-organisms which break down the organic materials.

Analysis

A simple one-dimensional analysis is considered to be appropriate for this study of aerated water jets. The following assumptions are made: steady flow, no coalescence or breakup of bubbles, no viscous dissipation effects, a thermally and calorically perfect gas, and no interaction

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between bubbles. Under certain circumstances these assumptions may break down, particularly under high bubble loading conditions. The pertinent algebraic and differential equations can be obtained by making the conventional conservation balances on a slice of the flowing mixture. References 2 and 3 outline some of the equations but the presentations are incomplete and, in Ref. 2, incorrect. Although most of the equations have been presented in Ref. 2, no solution has been reported and no study of the validity of the analysis has previously been attempted. In dimensional form the complete system of equations is

Bubble mass flow rate,

$$\dot{m}_B = \rho_B u_B \alpha A = \text{const} \tag{1}$$

Mixture mass flow rate,

$$(d/dy)(\dot{m}_B + \dot{m}_L) = E \tag{2}$$

where

$$\dot{m}_L = \rho_L u_L (1-\alpha) A$$

Mixture momentum conservation,

$$(\dot{m}_B/A)\, du_B/dy + (\dot{m}_L/A) = -dp/dy - F_s/A\Delta y - g[\rho_B\alpha + \rho_L(1-\alpha)] \tag{3}$$

Mixture energy conservation,

$$\frac{\dot{m}_B}{A}\left(c_{PB}\frac{dT_B}{dy} + u_B\frac{du_B}{dy}\right) + \frac{\dot{m}_L}{A}\left(c_{PL}\frac{dT_L}{dy} + u_L\frac{du_L}{dy}\right) + \frac{\dot{m}_B + \dot{m}_L}{A}g = \dot{q} \tag{4}$$

Liquid state relation,

$$\rho_L = \rho_L(T_L) \tag{5}$$

Bubble state relation,

$$\rho_B = \frac{p + 4\sigma/D_B}{R_B T_B} \tag{6}$$

Bubble force balance,

$$(\rho_B + k\rho_L) u_B\, du_B/dy - k\rho_L u_B\, du_L/dy = 3/4\, C_D \rho_L (u_L - u_B)|u_L - u_B|/D_B + g(\rho_L - \rho_B) - dp/dy + f \tag{7}$$

Bubble energy balance,

$$\rho_B u_B c_{pB}\, dT_B/dy + \rho_B u_B^2\, du_B/dy + \rho_B u_B g = 6h_s/D_B(T_L - T_B) \tag{8}$$

Bubble mass balance,

$$\rho_B V_B = \text{const} \tag{9}$$

These equations are applicable to either a single bubble or a swarm, as long as there is no interaction. The area A is regarded as an effective cross section area if the problem is one involving a jet of fluid, or can be an actual area in the case where the flow is confined (e.g., nozzle flow). The entrainment rate E must be specified for jet flows and is zero for confined flows; in the latter case, $\dot{m}_L$ = const. The shear force F_s, which acts on the nozzle walls, is zero for jet flows and can be computed as a function of the flow Reynolds number for confined flows. It is assumed that the difference between the bubble pressure and the liquid pressure is small and that an average pressure acts on the flow. Only in the bubble state relation is the surface tension taken into account. The added mass effect is included in the bubble force balance, but since there is no information available on the value of k for a deformable body, we use $k = 1/2$, applicable to a rigid sphere. The assumption that the bubble is a rigid sphere is not correct except at very small Reynolds numbers, but should be satisfactory compared to the assumptions necessary for a one-dimensional analysis. The term f in Eq. (7) is discussed in Ref. 4 and is a correction for viscous effects used in laminar flow.

Because it is assumed that there is no interaction between bubbles, wake effects are ignored. In a coflow where the bubble volume fraction is high, (say, >0.4), these wake effects must begin to be appreciable; thus, neglecting interaction should restrict the analysis to lower volume fractions.

Further assumptions with regard to the drag law and heat transfer law are necessary. Haberman and Morton[5] review the various modes of bubble motion and bubble shape as a function of Reynolds number (based on relative speed and bubble diameter). For $1 < Re < 10$, the bubble shape begins to deviate from that of a sphere; and for $Re \simeq 10$, turbulence starts to appear in the wake. For $10 < Re < 5000$, the drag coefficient is a function of Re, Weber number, liquid properties, turbulence level, impurities, etc. For $Re > 5000$ the spherical cap shape is observed. Obviously, a simple drag law is impossible to obtain, so some recourse to measured values of C_D is necessary. The data presented in Fig. 6 of Ref. 5 for bubbles moving in filtered water are used to input tubular values of C_D vs Re. Specifically, the drag coefficient decreases from a value of 2000 at $Re = 0.01$ to a minimum of 0.16 at $Re = 450$, and then increases to 2.6 at $Re = 3000$ at which point C_D is held constant.

There is a significant lack of data on heat transfer to bubbles in the Reynolds number range of interest, although a large amount of mass transfer data exists which may be related to heat-transfer coefficients under certain special conditions. The crude assumption of rigid spheres is used in the present analysis and Soo's[3] expression for h_s is incorporated

$$Nu = 2 + 0.459\, Re^{0.55} Pr_L^{0.33}$$

where

$$Nu = h_s D_B/\kappa_L$$

$$Re = |u_B - u_L| D_B/\nu_L$$

The only data on bubble coflow systems (of either the jet type or nozzle type) which could be obtained was that of Muir and Eichhorn[2] in which a water-air bubble mixture flowed in a converging, diverging nozzle. Two test conditions were reported, one for bubble volume fraction $\alpha = 0.1$ at the nozzle throat and one for $\alpha = 0.6$. Both of these cases are simulated by solving Eqs. (1-9) numerically. It is felt that the simulation of the Muir-Eichhorn data provides a good test of the one-dimensional analysis because of the large accelerations involved and the wide range of bubble loading conditions used. Further experimental work on bubble plume flows must be completed before a satisfactory test of the analysis for jetlike flows can be made.

As Muir and Eichhorn point out, there is no significant variation in water temperature from the plenum to the diffuser, so T_L is assumed constant, and ρ_L = const, thus removing Eqs. (4) and (5) from consideration. Also, the surface tension in Eq. (6) is excluded since the bubble diameter would have to be less than 3 μ for a significant effect in the Muir-Eichhorn tests. Other terms in Eqs. (7) and (8) may be dropped as a result of an order of magni-

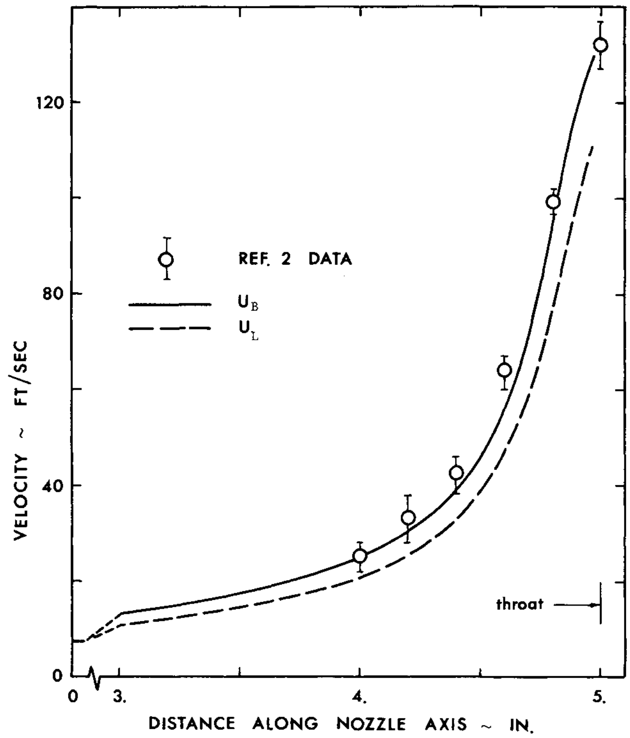

Fig. 1. Bubble and water velocity distributions for low bubble loading, $\alpha_{THROAT} = 0.1$.

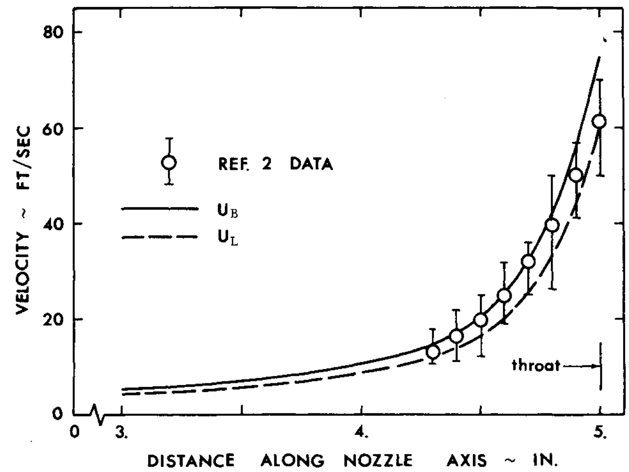

Fig. 2. Bubble and water velocity distributions for large bubble loading, $\alpha_{THROAT} = 0.6$.

tude analysis but are retained in the numerical solution because they introduce no complications.

The remaining four algebraic relations and three ordinary nonlinear differerential equations are to be solved for the variables u_B, ρ_B, α, D_B, T_B, u_L, and p as a function of y. First, all quantities are nondimensionalized by their initial values at the plenum conditions. Then three ordinary nonlinear differential equations are obtained in terms of derivatives of u_B, p, and α in the following form (where the β_i are the elements of the coefficient matrix)

$$\begin{bmatrix} \beta_1 & \beta_2 & \beta_3 \\ \beta_4 & \beta_5 & \beta_6 \\ \beta_7 & \beta_8 & \beta_9 \end{bmatrix} \begin{bmatrix} u_B' \\ p' \\ \alpha' \end{bmatrix} = \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \end{bmatrix} \tag{10}$$

Solutions for u_B, p', and α' are obtained by use of Cramer's rule, and are expressible as functions of only the dependent variables

$$du_B/dy = f_1(u_B, p, \alpha)$$

$$dp/dy = f_2(u_B, p, \alpha)$$

$$d\alpha/dy = f_3(u_B, p, \alpha)$$

This system of three nonlinear differential equations has been solved by Hamming's predictor-corrector scheme for the given initial conditions of each test. The remaining four parameters—u_L, D_B, T_B, ρ_B—are obtained from algebraic relations.

Results and Discussion

Some results of the numerical solutions are shown in Figs. 1 and 2 where air bubble velocity and water velocity are plotted in each figure and are to be compared with the data which Muir and Eichhorn obtained for bubble velocity. For Fig. 1, the initial value of bubble loading α in the plenum is 0.0179, and the simulation is quite good as expected for low bubble loadings. Reference 2 gives the value of α at the throat as 0.1; the predicted value is extrapolated to be 0.11. The simulation in Fig. 2 is only fair, but this was also expected, since the bubble loading at the throat was measured in Ref. 2 to be 0.6 (predicted value 0.56). The photographs shown in Ref. 2 reveal that the flow near the nozzle throat for the high bubble loading condition was almost annular; i.e., a slug of air surrounded by a film of water. Under these conditions, the equations used are not expected to accurately represent the experimental situation.

Note that in Fig. 1 the solution does not quite reach the throat. This is because the determinant of the coefficient matrix of Eq. (10) becomes very small before the physical throat is reached. At this point, the solution is singular and another approach must be used to continue the analysis. This has not been attempted in the present study.

Several tests have been made to study the effects of varying k, the added mass coefficient and F_s, the shear force relation. There is approximately a 6% change in the bubble velocity for a factor of four variation in magnitude of both k and F_s. Also, tests on varying C_D by a factor of three have been carried out, and the effect on the velocity distribution is relatively small (about 10%).

The one-dimensional simulation is satisfactory for nozzle-type coflows and provides a suitable prediction of all pertinent dependent variables. The scheme is currently being studied for jetlike flows where entrainment must be considered. There is a lack of data for heat-transfer coefficients of deformable bubbles, and a lack of accurate drag data in the large Reynolds number range. Because of these limitations a program for experimental and analytical study of heat transfer to bubbles has been initiated. Considering the possible importance of bubble coflows, more experimental work is required.

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